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ABSTRACT

This paper extends the empirical investigation of the shape of wage-profit curves to the case of joint production using data from the Supply and Use Tables of the Finnish economy (for the years 1995 through 2004). It is found that (i) the considered systems do not have the usual properties of single-product systems; and (ii) the monotonicity of the wage-profit curves depends on the adopted normalization condition.

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1. INTRODUCTION

Sraffa's claim that a production technique which is cost-minimizing at two disconnected ranges of the rate of profits could be inferior in between these ranges (Sraffa, 1960, ch. 12) was one of the central issues in the famous 'Cambridge' capital controversy.[1] This phenomenon, which is known as the 'reswitching' of techniques, implies that basic propositions of neoclassical theory, such as the inverse relation between the value of capital per worker and the rate of profits (i.e., 'capital deepening'), are invalid. Furthermore, the existence of reswitching implies that an analysis in terms of Marxian 'labour values' is inconsistent with the results obtained in terms of prices.[2] Although the theoretical possibility of reswitching and/or 'reverse capital deepening' cannot be denied,[3] there is a significant number of empirical studies that cast doubt on their importance in the actual world.[4] More specifically, these studies investigate the shape of wage-profit ($w-r$ hereafter) curves constructed from input-output data for different years. The central conclusions are that, in actual economies, the $w-r$ curves (i) are 'nearly' linear; and (ii) do not display many curvatures.[5] On the basis of these findings, it is usually argued that the so-called 'paradoxes' in capital theory, i.e., reswitching and reverse capital deepening, are empirically unimportant.[6] The same line of argument suggests that although the basic propositions of neoclassical theory and Marxian labour theory of value are not true in general, they appear to be good approximation to reality (see, *e.g.*, Petrović (1991, p. 108)).

[1] For an extensive analysis of the controversy, see Kurz and Salvadori (1995, ch. 14).

[2] For an exhaustive Sraffa-based critique of the Marxian labour theory of value, see Steedman (1977).

[3] It should be noted that reswitching is a sufficient but not necessary condition for reverse capital deepening.

[4] See Krelle (1977), Ochoa (1984, 1989, 1992), Petrović (1991), da Silva (1991), da Silva and Rosinger (1992), Tsoulfidis and Maniatis (2002), Tsoulfidis and Rieu (2006), inter alia.

[5] For a theoretical investigation of the shape of the $w-r$ curves, see Schefold (2008).

[6] It is well known that if the $w-r$ curves are linear, then reverse capital deepening and reswitching of techniques are impossible. It should be noted, however, that Mainwaring and Steedman (2000) found that on the basis of a two-sector Sraffian model the highest probability of reswitching is observed in the case of $w-r$ curves of relatively low concavity. Thus, it cannot be claimed that there is a straightforward relationship between probability of reswitching and curvature. Moreover, Kurz and Salvadori (1995, p. 450) note that the methodology followed by the aforesaid studies cannot empirically prove/disprove reswitching, because the calculated $w-r$ curves represent different technologies, whilst the reswitching argument 'refers to the technical knowledge at a given moment of time' (ibid.). For an alternative approach to investigate the existence of reswitching in the real world that overcomes to a certain extent the shortcomings of the traditional procedure, see Han and Schefold (2006).

To our knowledge, all the empirical studies that investigate the shape of $w-r$ curves are based on data from Symmetric Input-Output Tables (SIOT). As is well known, the SIOT can be derived from the 'System of National Accounts' framework of the Supply and Use Tables (SUT) (see, e.g., United Nations, 1999, chs 2-4 and Eurostat, 2008, ch. 11), introduced in 1968 (United Nations, 1968, ch.3).[7] Given that in the SUT (SIOT) there are (are no) industries that produce more than one commodity and (nor) commodities that are produced by more than one industry, it follows that the SUT (SIOT) could be considered as the counterpart of a joint production (single-product) system à la v. Neumann/Sraffa.[8] Nevertheless, since joint production is the empirically relevant case (see Steedman, 1984; Faber et al., 1998), SUT constitute, doubtless, a more realistic 'picture' of the actual economic system than SIOT.

The purpose of this paper is to estimate, in terms of the usual 'square' linear model of production (for a closed economy with circulating capital and homogeneous labour), the $w-r$ curves associated with the SUT of the Finnish economy (for the years 1995 through 2004).[9] It is important to note that we decided to use Finland's SUT mainly because Statistics Finland provides all the required data for such an investigation.[10]

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 explores whether the actual systems under consideration satisfy certain conditions that ensure that they behave like single-product systems. Section 4 presents the $w-r$ curves of the Finnish economy and examines their monotonicity, curvature and linearity. Section 5 concludes the paper.

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[7] For a review of the methods, used to convert the SUT into SIOT, see, *e.g.*, ten Raa and Rueda-Cantuche (2003, pp. 441-447). Amongst the various available methods, the so-called 'Commodity Technology Assumption' is the only one that fulfils a set of important properties of the input-output analysis (see Jansen and ten Raa, 1990). However, the 'Commodity Technology Assumption' is possible to generate economically insignificant results, i.e., negative elements in the input-output matrix. For a critical review of the various procedures proposed to overcome this inconsistency, see ten Raa and Rueda-Cantuche (2005).

[8] See, e.g., Flaschel (1980, pp. 120-121), Bidard and Erreygers (1998, pp. 434-436) and Lager (2007). It has to be noted, however, that some of the 'joint' products that appear in the SUT may result from statistical classification and, therefore, they do not correspond with the notion of joint production (see, e.g., Semmler, 1984, pp. 168-169; United Nations, 1999, p. 77).

[9] It goes without saying that the SUT are not necessarily square (see, *e.g.*, United Nations (1999, p. 86, §4.41) and Eurostat (2008, p. 295, §11.1), whilst for the relevant theoretical discussion, see, *e.g.*, Steedman (1976a), Schefold (1978a), and Bidard (1986a, 1997)).

[10] See the Appendix for the available input-output data as well as the construction of relevant variables.

2. THE ANALYTIC FRAMEWORK

Assume a closed capitalist economy, which produces n commodities by n linear processes of pure joint production, i.e., a 'square', profitable and productive system, and in which commodity prices deviate from the prices of production. Homogeneous labour is the only primary input and there is only circulating capital, whilst labour is not an input to the household sector. Moreover, the net product is distributed to profits and wages that are paid at the beginning of the common production period.[11] Finally, we assume as given the technical conditions of production, that is, the triplet $\{\mathbf{B}, \mathbf{A}, \mathbf{a}\}$, where $\mathbf{B}$ represents the $n \times n$ Make matrix, $\mathbf{A}$ the $n \times n$ Use matrix (both $\mathbf{B}$ and $\mathbf{A}$ are expressed in physical terms), and $\mathbf{a}^{\mathrm{T}}$ the $1 \times n$ vector of employment levels process by process ('T' is the sign for transpose). On the basis of these assumptions, the vector of production prices, $\mathbf{p}$, is defined by the following equations

$$\mathbf{p}^{\mathrm{T}}\mathbf{B} = (1+r)(\mathbf{p}^{\mathrm{T}}\mathbf{A} + w\mathbf{a}^{\mathrm{T}}) \tag{1}$$

$$\mathbf{p}^{\mathrm{T}}\mathbf{z} = 1 \tag{2}$$

where r is the uniform rate of profits, w the money wage rate and $\mathbf{z}$ the standard of value or numéraire. Provided that $[\mathbf{B} - (1+r)\mathbf{A}]$ is non-singular, (1) and (2) entail that

$$w = [(1+r)\mathbf{a}^{\mathrm{T}}\mathbf{C}(r)\mathbf{z}]^{-1} \tag{3}$$

where $\mathbf{C}(r) \equiv [\mathbf{B} - (1+r)\mathbf{A}]^{-1}$. Equation (3) gives a $w-r$ curve for this economy. By contrast with the case of single-product systems, $\mathbf{C}(r)$ can contain one or more negative elements. In the case where $[\mathbf{B} - \mathbf{A}]^{-1} > \mathbf{0}$ ($[\mathbf{B} - \mathbf{A}]^{-1} \geq \mathbf{0}$), the system $\{\mathbf{B}, \mathbf{A}\}$ is called 'all-engaging' ('all-productive') and is characterized by $\mathbf{C}(r) > \mathbf{0}$ ($\mathbf{C}(r) \geq \mathbf{0}$) for $0 \leq r \leq R$, where R is the only positive root of $\det[\mathbf{B} - (1+\mathrm{r})\mathbf{A}] = 0$ associated with a positive eigenvector.[12] Therefore, when the system $\{\mathbf{B}, \mathbf{A}\}$ is all-engaging (all-productive), it holds $\mathbf{p} > \mathbf{0}$ ($\mathbf{p} \geq \mathbf{0}$) for $0 \leq r \leq R$ and the $w-r$ curve is downward sloping. On the other hand, when $[\mathbf{B} - \mathbf{A}]^{-1}$ contains negative elements, nothing guarantees the existence of an interval of r in which prices are (semi-)

[11] Following Classical economists and Marx, we hypothesize that wages are paid ante factum. For the general case, see Steedman (1977, pp. 103-105).

[12] The concept of all-engaging (all-productive) systems, introduced by Schefold, is of significant importance since it corresponds with systems that retain all the essential properties of indecomposable (decomposable) single-product systems (see Schefold, 1971, pp. 34-35, 1978b, Kurz and Salvadori, 1995, pp. 238-240, and Bidard, 1996).

positive.[13] Moreover, even if such an interval exists, the monotonicity of the $w-r$ curve is a priori unknown and can depend on the adopted normalization condition.[14] In that case it is important to study whether the 'labour commanded' prices, $\mathbf{P}\ (\equiv \mathbf{p}/w)$, are directly related with the rate of profits, because iff each element of the vector of labour commanded prices is a strictly increasing function of the profit rate, then the $w-r$ curve is decreasing irrespective of the numéraire chosen.[15] As is well known, some labour commanded prices are inversely related with the rate of profits iff there exists a non-negative linear combination of processes that yields a greater net net output than a non-negative linear combination of the remaining ones, whilst the input values of the former combination are lower (see Filippini and Filippini, 1982, pp. 389-390; Salvadori and Steedman, 1988, p. 181).

Finally, it should be stressed that any 'complication' related to joint production, *i.e.*, non-squareness, inconsistency or non-unique economically

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[13] Some prices are negative at given profit rate iff there exists a non-negative linear combination of processes that yields a greater net net output (i.e., gross output minus $(1+r)$ times the production inputs) than a non-negative linear combination of the remaining ones (see Filippini and Filippini, 1982, pp. 387-388; Salvadori and Steedman, 1988, p. 179).

[14] For the implications of upward sloping $w-r$ curves in economic theory, see Steedman (1982), d' Autume (1988) and Mariolis (2004, 2008).

[15] From equation (1) it follows

$$\mathbf{P}^{\mathrm{T}}\mathbf{B} = (1+r)(\mathbf{P}^{\mathrm{T}}\mathbf{A} + \mathbf{a}^{\mathrm{T}}) \tag{1a}$$

or, solving for $\mathbf{P}^{\mathrm{T}}$,

$$\mathbf{P}^{\mathrm{T}} = (1+r)\mathbf{a}^{\mathrm{T}}\mathbf{C}(r)$$

Differentiating with respect to r we get

$$\mathbf{P}^{\mathrm{T}\prime} \equiv d\mathbf{P}^{\mathrm{T}}/dr = (1+r)\mathbf{f}^{\mathrm{T}}(r) + \mathbf{a}^{\mathrm{T}}\mathbf{C}(r)$$

where $\mathbf{f}^{\mathrm{T}}(r) \equiv \mathbf{a}^{\mathrm{T}}\mathbf{C}(r)\mathbf{A}\mathbf{C}(r)$. If wages were to be paid at the end of the common production period, equation (1a) would have to be replaced by

$$\mathbf{P}_{\mathrm{e}}^{\mathrm{T}}\mathbf{B} = (1+r)\mathbf{P}_{\mathrm{e}}^{\mathrm{T}}\mathbf{A} + \mathbf{a}^{\mathrm{T}} \tag{1b}$$

or, solving for $\mathbf{P}_{\mathrm{e}}^{\mathrm{T}}$,

$$\mathbf{P}_{\mathrm{e}}^{\mathrm{T}} = \mathbf{a}^{\mathrm{T}}\mathbf{C}(r)$$

Differentiating with respect to r we obtain

$$\mathbf{P}_{\mathrm{e}}^{\mathrm{T}\prime} \equiv d\mathbf{P}_{\mathrm{e}}^{\mathrm{T}}/dr = \mathbf{f}^{\mathrm{T}}(r)$$

Thus, it follows that if an individual component of $\mathbf{P}^{\mathrm{T}\prime}$ is negative, then the corresponding element of $\mathbf{P}_{\mathrm{e}}^{\mathrm{T}\prime}$ will also be negative. In other words, if a $w-r$ curve is increasing when wages are paid ante factum, it will also be increasing in the case where the wages are paid post factum. For a relevant discussion, see Bidard (1996, pp. 326-327). I am grateful to Theodore Mariolis for an enlightening discussion on this point.

significant solution for $(r, \mathbf{p})$, can be adequately handled on the basis of general joint production models inspired by v. Neumann (1945) and Sraffa (1960).[16]

3. EMPIRICAL INVESTIGATION

The application of the previous analysis to the SUT of the Finnish economy (for the years 1995 through 2004) gives the following results:[17]

(i). The matrices $[\mathbf{B}-\mathbf{A}]$ are non-singular and, therefore, invertible.

(ii). The matrices $[\mathbf{B}-\mathbf{A}]^{-1}$ contain negative elements. Consequently, the systems under consideration are not all-productive and, therefore, they do not have the properties of a single-product system.[18]

The next issue that comes up is whether the systems under consideration are 'r-all-engaging', i.e., characterized by $\mathbf{C}(r) > \mathbf{0}$ for some $r > -1$. As is well known, $\mathbf{C}(r) > \mathbf{0}$ is a sufficient condition for the existence of an interval of r, in which a joint production system retains all the essential properties of indecomposable single-product systems (see Schefold, 1971, p. 35; 1978b; Bidard, 1996).[19] The investigation can be based on the following theorem (Bidard, 1996, p. 328): Consider the eigensystems associated with the pair $\{\mathbf{B}, \mathbf{A}\}$, namely

$$\lambda \mathbf{B}\mathbf{x} = \mathbf{A}\mathbf{x} \tag{4}$$

$$\lambda \mathbf{y}^{\mathrm{T}}\mathbf{B} = \mathbf{y}^{\mathrm{T}}\mathbf{A} \tag{5}$$

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[16] For a detailed exposition of the v. Neumann/Sraffa-based analysis and the connection between the works of v. Neumann and Sraffa, see Kurz and Salvadori (1995, ch. 8 and pp. 421-426; 2001) and Bidard (1997).

[17] Mathematica 7.0 is used in the calculations, whilst the precision in internal calculations is set to 16 digits. All the analytical results are available on request from the author.

[18] It may also be added that Mariolis and Soklis (2010) found that the systems associated with the SUT of the French (for the years 1995 and 2005), German (for the years 2000 and 2005) and Greek (for the years 1995 and 1999) economies are not all-productive. Furthermore, empirical research from the author of this paper, based on the SUT of the Danish (for the years 2000 and 2004), German (for the years 1997-1999 and 2001-2005), Greek (for the years 1996-1998), F.Y.R.O.M. (for the year 2005), Hungarian (for the years 2001-2004), Japanese (for the years 1970, 1975, 1980, 1985, 1990, 1995, 2000), Slovenian (for the years 2002-2005), Swedish (for the years 1995-2005) and USA (for the years 1998-2005) economies, yielded the same result.

[19] It is important to note that this attribute of the considered systems is independent of the values of the variables of the distribution of income. Furthermore, since the matrices $[\mathbf{B}-\mathbf{A}]^{-1}$ contain negative elements, it follows that the systems under consideration can be r-all-engaging only for some $r > 0$ (ibid.).

The system $\{\mathbf{B}, \mathbf{A}\}$ is r-all-engaging iff there exist $(\lambda, \mathbf{x}, \mathbf{y}) > \mathbf{0}$, where $\mathbf{x}$ is determined up to a factor.[20]

The estimation of the characteristic values and vectors associated with the pairs $\{\mathbf{B}, \mathbf{A}\}$ of the Finnish economy gives the following results: The eigensystems for the years 1995 through 2004 have 16 (1995), 14 (1996), 24 (1997), 15 (1998), 16 (1999), 18 (2000), 22 (2001), 15 (2002), 18 (2003) and 20 (2004) positive and simple eigenvalues, respectively. However, there are no positive left and right eigenvectors and, therefore, the considered systems are <u>not</u> r-all-engaging. Nevertheless, it is worth noting that the subdominant (dominant) eigenvalue of the pair $\{\mathbf{B}, \mathbf{A}\}$ for the year 2001 (2002) is positive ($\lambda_2 \simeq 0.77$ (2001), $\lambda_1 \simeq 0.79$ (2002)) and associated with a positive left eigenvector. Since the eigenequation (5) corresponds to the system of production prices for a zero wage, one may, speaking somewhat loosely, conceive $R_2 = (\lambda_2^{-1} - 1) \simeq 0.30$ ($R_1 = (\lambda_1^{-1} - 1) \simeq 0.27$) as a <u>meaningful</u> theoretical maximum rate of profits for the year 2001 (2002).[21]

As mentioned above, $\mathbf{C}(r) > \mathbf{0}$ is a sufficient condition for the existence of an interval of r, in which production prices are positive and the $w - r$ curves downward sloping. However, even if $\mathbf{C}(r)$ contains negative elements, it is entirely possible for $\mathbf{a}^{\mathrm{T}}\mathbf{C}(r) \geq \mathbf{0}^{\mathrm{T}}$ (or, equivalently, $\mathbf{P} \geq \mathbf{0}$) to hold for some r. In what follows we investigate whether such intervals of the profit rate exist in the Finnish economy. Calculations are performed by varying profit rate from zero to one ($0 \leq r \leq 1$) with step equal to 0.01.[22] The results of the investigation are reported in Table 1.

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[20] In that case $\lambda^{-1} - 1$ represents the maximum possible rate of growth (and profits), as defined by v. Neumann (1945), $\mathbf{y}^{\mathrm{T}}$ the associated price vector, and $\mathbf{x}$ the associated intensity vector or, alternatively, the intensity vector of Sraffa's (1960, ch. 8) 'Standard system'.

[21] It may also be added that the dominant eigenvalue of the pair $\{\mathbf{B}, \mathbf{A}\}$ for the year 2001 is positive ($\lambda_1 \simeq 0.924$) and is associated with non-positive right and left eigenvectors. Thus, it follows that the production prices would be infinite at $R_1 = (\lambda_1^{-1} - 1) \simeq 0.082$ (Sraffa, 1960, §64; Bidard, 1986b). I am grateful to Theodore Mariolis for pointing this out to me.

[22] Since we have found a meaningful maximum rate of profits, we restrict our investigation to the interval $0 \leq r < 0.30$ ($0 \leq r < 0.27$) for the year 2001 (2002). On the other hand, the choice of the interval of r, in which we investigate the systems for the remaining years, is conventional.

Table 1. Intervals of the profit rate in which the 'labour commanded' prices are positive; Finnish economy, 1995-2004.

Year	$\mathbf{a}^{\mathrm{T}}\mathbf{C}(r) > \mathbf{0}^{\mathrm{T}}$
1995	$0 \le r \le 0.14$
1996	$0 \le r < 0.01$
1997	$0 \le r \le 0.11$
1998	$0 \le r \le 0.05$
1999	-
2000	$0.03 \le r \le 0.23$
2001	$0.09 \le r \le 0.29$
2002	$0.25 \le r \le 0.26$
2003	-
2004	-

It is found that there exists an interval of r, such that $\mathbf{a}^{\mathrm{T}}\mathbf{C}(r) > \mathbf{0}^{\mathrm{T}}$, for the years 1995 through 1998 and 2000 through 2002. The larger interval is found for the years 2000 and 2001 ($0.03 \le \mathrm{r} \le 0.23$ and $0.09 \le \mathrm{r} \le 0.29$, respectively), whilst the smallest interval is found for the year 1996 ($0 \le \mathrm{r} < 0.01$).[23]

In the next section we estimate the $w-r$ curves for the years 1995, 1997, 2000 and 2001 of the Finnish economy associated with the intervals of the uniform rate of profits reported in Table 1.[24]

[23] It goes without saying that the vector of 'additive labour values' (for this concept, see Steedman, 1975, 1976b) is positive only in the case where $\mathbf{a}^{\mathrm{T}}\mathbf{C}(0) > \mathbf{0}^{\mathrm{T}}$. Thus, the additive labour values of the Finnish economy are positive only for the years 1995 through 1998. Furthermore, it is interesting to note that the 'actual' $(r, \mathbf{p})$, i.e., those that correspond to the 'actual' real wage rate (estimated on the basis of the available input-output data; see the Appendix), are economically significant only for the years 2000 through 2002. For an investigation of the conditions that guarantee the existence of (semi-)positive solution for $(r, \mathbf{p})$, with the real wage rate exogenously given as a fixed consumption basket, see Fujimoto and Krause (1988). Finally, for an empirical investigation of the relationships between prices and additive labour values of the French, German and Greek economies, see Mariolis and Soklis (2010).

[24] We exclude from our investigation the years 1996, 1998 and 2002 because the intervals of the uniform rate of profits, in which the systems give economically significant results, can be considered as quite 'small'.

4. THE WAGE-PROFIT CURVES OF THE FINNISH ECONOMY

The $w-r$ curves of the Finnish economy (for years 1995, 1997, 2000 and 2001) are obtained on the basis of relation (3), whilst we use as numéraires (i) each of the 57 commodities of the economy; (ii) the gross output; (iii) the net output; and (iv) the 'actual' real wage rate. Thus, we obtain 60 $w-r$ curves for each year. We examine (i) the monotonicity of the curves; (ii) the curvature of the downward sloping $w-r$ curves, i.e., whether these curves are convex or concave to the origin, and we detect the points where the curvature switches from convex to concave or vice versa; and (iii) the linearity of the downward sloping curves by using Pearson's correlation coefficient. Calculations are performed by varying profit rate from its 'minimum' to its 'maximum' with step equal to 0.001. The numerical results of the analysis are reported in Tables 2-3. Table 2 reports the results for the years 1995 and 1997, whilst Table 3 reports the results for the years 2000 and 2001. The first column of the tables indicates the CPA code of the commodity that is used as numéraire.[25] The second column reports the monotonicity of the $w-r$ curves. The symbol $\searrow (\nearrow)$ indicates that a curve is strictly decreasing (increasing) for the whole interval of r, whilst $\nearrow\searrow(\bullet)$ indicates that a curve is alternately increasing and decreasing, and $\bullet$ indicates the value of r where the monotonicity is reversed.[26] The third column reports the curvature of the $w-r$ curves. The abbreviation CX (CV) denotes that a curve is convex (concave) for the whole interval of r, whilst CX/CV$(\bullet)$ (CV/CX$(\bullet)$) denotes that a curve switches from convex (concave) to concave (convex), and $\bullet$ indicates the value of r where the first concave (convex) point is detected. Also, CV/CX$(\bullet)$/CV$(\circ)$ denotes that a curve switches from concave to convex at $r=\bullet$ and re-switches from convex to concave at $r=\circ$.[27] Finally, the fourth column reports the absolute values of Pearson's correlation coefficient, $|\rho|$, between w and r.

[25] The nomenclature of each commodity and the corresponding CPA code is reported in the Appendix, Table A1.

[26] It is interesting to note that it has not been found any case where the monotonicity of a $w-r$ curve is alternately decreasing and increasing.

[27] It is worth noting that it has not been found any case where a $w-r$ curve switches from convex to concave and then re-switches to convex.

Table 2. Shape of the wage-profit curves; Finnish Economy, 1995 and 1997.

Numéraires-commodities (CPA)	Monotonicity of $w-r$ curves; 1995	Curvature of $w-r$ curves; 1995	Correlation coefficient between w and r $\lvert\rho\rvert$; 1995	Monotonicity of $w-r$ curves; 1997	Curvature of $w-r$ curves; 1997	Correlation coefficient between w and r $\lvert\rho\rvert$; 1997
01	↘	CX	99.91%	↘	CX	99.94%
02	↘	CX	99.96%	↘	CX	99.98%
05	↘	CX	99.93%	↘	CX	99.96%
10	↘	CX	99.93%	↘	CX	99.95%
13	↘	CX	99.94%	↘	CX	99.95%
14	↘	CX	99.94%	↘	CX	99.97%
15	↘	CX	99.91%	↘	CX	99.94%
16	↘	CX	99.93%	↘	CX	99.97%
17	↘	CX	99.95%	↘	CX	99.98%
18	↘	CX	99.95%	↘	CX	99.97%
19	↘	CX	99.94%	↘	CX	99.97%
20	↘	CX	99.95%	↘	CX	99.97%
21	↘	CX	99.92%	↘	CX	99.96%
22	↘	CX	99.95%	↘	CX	99.97%
$23 \oplus 11$	↘	CX	99.81%	↘	CX	99.89%
24	↘	CX	99.93%	↘	CX	99.96%
25	↘	CX	99.95%	↘	CX	99.98%
26	↘	CX	99.96%	↘	CX	99.97%
27	↘	CX	99.87%	↘	CX	99.90%
28	↘	CX	99.96%	↘	CX	99.98%
29	↘	CX	99.95%	↘	CX	99.96%
30	↘	CX	99.86%	↘	CX	99.90%
31	↘	CX	99.94%	↘	CX	99.97%
32	↘	CX	99.91%	↘	CX	99.94%
33	↘	CX	99.95%	↘	CX	99.97%
34	↘	CX	99.94%	↘	CX	99.97%
35	↘	CX	99.95%	↘	CX	99.97%
36	↘	CX	99.96%	↘	CX	99.98%
37	↗	-	-	↗	-	-
40	↘	CX	99.95%	↘	CX	99.98%
41	↘	CX	99.97%	↘	CX	99.98%
45	↘	CX	99.96%	↘	CX	99.98%
50	↘	CX	99.97%	↘	CX	99.98%
51	↘	CX	99.97%	↘	CX	99.98%
52	↘	CX	99.97%	↘	CX	99.98%
55	↘	CX	99.96%	↘	CX	99.97%
60	↘	CX	99.98%	↘	CX/CV (7.5%)	100.00%
61	↘	CX	99.95%	↘	CX	99.98%
62	↘	CX	99.96%	↘	CX	99.98%
63	↘	CX	99.95%	↘	CX	99.97%

64	↘	CX	99.97%	↘	CX	99.98%
65	↘	CX	99.97%	↘	CX	99.98%
66	↘	CX	99.95%	↘	CX	99.97%
67	↘	CX	99.96%	↘	CX	99.98%
70	↘	CX	99.92%	↘	CX	99.95%
71	↘	CX	99.96%	↘	CX	99.98%
72	↘	CX	99.97%	↘	CX	99.98%
73	↘	CX	99.97%	↘	CX	99.98%
74	↘	CX	99.97%	↘	CX	99.98%
75	↘	CX	99.97%	↘	CX	99.98%
80	↘	CX	99.97%	↘	CX	99.98%
85	↘	CX	99.97%	↘	CX	99.98%
90	↘	CX	99.96%	↘	CX	99.98%
91	↘	CX	99.97%	↘	CX	99.98%
92	↘	CX	99.97%	↘	CX	99.98%
93	↘	CX	99.98%	↘	CX	99.99%
95	↘	CX	99.94%	↘	CX	99.96%
REAL WAGE RATE	↘	CX	99.96%	↘	CX	99.98%
GROSS OUTPUT	↘	CX	99.93%	↘	CX	99.97%
NET OUTPUT	↘	CX	99.94%	↘	CX	99.97%

Table 3. Shape of the wage-profit curves; Finnish Economy, 2000 and 2001.

Numéraires-commodities (CPA)	Monotonicity of $w-r$ curves; 2000	Curvature of $w-r$ curves; 2000	Correlation coefficient between w and r $\lvert\rho\rvert$; 2000	Monotonicity of $w-r$ curves; 2001	Curvature of $w-r$ curves; 2001	Correlation coefficient between w and r $\lvert\rho\rvert$; 2001
01	↘	CX/CV (12.1%)	99.67%	↘	CV/CX (10.3%)/CV (15.5%)	98.46%
02	↘	CX/CV (6.7%)	98.15%	↘	CV/CX (9.8%)/CV (12%)	94.52%
05	↘	CX/CV (10.6%)	99.45%	↘	CV/CX (9.9%)/CV (16.2%)	97.67%
10	↘	CX/CV (9.4%)	99.28%	↘	CV/CX (10.2%)/CV (15.1%)	97.40%
13	↘	CV/CX (3.2%)/CV (5.9%)	98.78%	↘	CV/CX (11.3%)/CV (12.9%)	96.20%
14	↘	CX/CV (9.2%)	99.43%	↘	CX/CV (14.4%)	98.02%
15	↘	CX/CV (12.2%)	99.67%	↘	CV/CX (10.3%)/CV (16.3%)	98.18%
16	↘	CX/CV (9.7%)	99.28%	↘	CV/CX (9.9%)/CV (16.2%)	98.01%
17	↘	CX/CV (7.4%)	98.90%	↘	CV/CX (10.6%)/CV (12.2%)	96.70%
18	↘	CX/CV (8.6%)	98.93%	↘	CV/CX (10%)/CV (14.6%)	96.63%
19	↘	CX/CV (9.1%)	99.12%	↘	CV/CX (10.2%)/CV (14.7%)	96.78%
20	↘	CX/CV (8.8%)	99.04%	↘	CV/CX (12.2%)/CV (13.2%)	96.49%
21	↘	CV/CX (3.6%)/CV (10.0%)	99.44%	↘	CV/CX (10.4%)/CV (13.4%)	97.80%
22	↘	CV/CX (3.2%)/CV (12.7%)	99.77%	↘	CV/CX (9.8%)/CV (15.2%)	96.64%
$23 \oplus 11$	↘	CX/CV (21.2%)	99.51%	↘	CV/CX (9.3%)	99.33%
24	↘	CX/CV (9.1%)	99.44%	↘	CV/CX (11.2%)/CV (13.3%)	97.98%
25	↘	CX/CV (8.1%)	99.07%	↗ ↘ (9.2%)	-	-
26	↘	CV/CX (4.9%)/CV	99.10%	↗ ↘ (10.6%)	-	-

		(8.0%)				
27	↘	CX/CV (12.0%)	99.71%	↘	CV/CX (10.4%)/CV (16.9%)	98.58%
28	↘	CX/CV (9.1%)	99.30%	↘	CV/CX (11.2%)/CV (15.1%)	97.58%
29	↘	CX/CV (10.4%)	99.39%	↘	CV/CX (10.2%)/CV (17.3%)	97.84%
30	↘	CX	99.40%	↘	CV/CX (9.4%)/CV (19.5%)	99.41%
31	↘	CX/CV (11.3%)	99.53%	↘	CV/CX (10.8%)/CV (17.4%)	98.30%
32	↘	CX/CV (14.3%)	99.84%	↘	CV/CX (10%)/CV (20.5%)	99.20%
33	↘	CX/CV (6.0%)	98.92%	↘	CV/CX (10.3%)/CV (17.2%)	97.95%
34	↘	CX/CV (9.7%)	99.26%	↘	CV/CX (11.3%)/CV (15.6%)	97.20%
35	↘	CX/CV (10.4%)	99.38%	↘	CV/CX (10.8%)/CV (17.1%)	97.98%
36	↘	CX/CV (8.7%)	99.04%	↘	CV/CX (11.5%)/CV (14.0%)	96.73%
37	↗	-	-	↗ ↘ (24.9%)	-	-
40	↘	CX/CV (6.3%)	99.50%	↘	CV	99.03%
41	↘	CX/CV (4.8%)	98.42%	↘	CV	95.36%
45	↘	CV/CX (3.4%)/CV (8.5%)	98.97%	↘	CV	96.74%
50	↘	CV	98.30%	↘	CV	95.41%
51	↘	CV	98.44%	↘	CV	95.94%
52	↘	CV	97.57%	↘	CV	94.32%
55	↘	CX/CV (8.0%)	98.92%	↘	CV/CX (10.4%)/CV (14.4%)	96.55%
60	↘	CV	98.19%	↘	CV	95.34%
61	↘	CX/CV (6.9%)	99.16%	↘	CV	97.87%
62	↘	CV/CX (3.2%)/CV (4.5%)	99.33%	↘	CV	98.56%
63	↘	CX/CV (6.6%)	98.67%	↘	CV/CX (10.7%)/CV (12.7%)	96.15%
		CX/CV			CV/CX	

64	↘	(7.3%)	98.54%	↘	(11.2%)/CV (13.7%)	95.74%
65	↘	CV	97.51%	↘	CV	94.57%
66	↘	CX/CV (4.9%)	97.10%	↘	CV/CX (9.4%)/CV (15.4%)	95.94%
67	↘	CX/CV (4.9%)	97.85%	↘	CV/CX (10.0%)/CV (15.1%)	95.77%
70	↘	CX/CV (9.1%)	99.25%	↘	CV/CX (11.2%)/CV (15.4%)	97.51%
71	↘	CV	99.03%	↘	CV/CX (11.4%)/CV (13.2%)	96.43%
72	↘	CV/CX (3.2%)/CV (3.6%)	97.34%	↘	CV/CX (10.0%)/CV (14.3%)	95.22%
73	↘	CX/CV (5.2%)	97.32%	↘	CV/CX (10.1%)/CV (14.3%)	94.99%
74	↘	CV	98.18%	↘	CV/CX (10.3%)/CV (13.5%)	95.63%
75	↘	CX/CV (7.2%)	98.21%	↘	CV/CX (10.2%)/CV (13.9%)	94.74%
80	↘	CX/CV (7.8%)	98.15%	↘	CV/CX (9.8%)/CV (14.6%)	94.28%
85	↘	CX/CV (7.4%)	97.99%	↘	CV/CX (10.4%)/CV (14.0%)	93.96%
90	↘	CV	98.92%	↘	CV/CX (10.9%)/CV (12.1%)	96.24%
91	↘	CX/CV (6.6%)	97.97%	↘	CV/CX (9.9%)/CV (12.9%)	93.88%
92	↘	CX/CV (7.0%)	98.40%	↘	CV/CX (10.3%)/CV (13.4%)	95.05%
93	↘	CV	99.16%	↘	CV	96.27%
95	↘	CX	99.89%	↘	CX	99.90%
REAL WAGE RATE	↘	CV/CX (3.2%)/CV (5.5%)	98.72%	↘	CV	96.33%
GROSS OUTPUT	↗ ↘ (3.4%)	-	-	↗ ↘ (12.2%)	-	-
NET OUTPUT	↘	CV	97.11%	↗ ↘ (10.1%)	-	-

From the numerical results of our analysis, we arrive at the following conclusions:

Monotonicity

(i). The curves for the years 1995 and 1997 are strictly decreasing, except for the case where the product 37 (‘Secondary raw materials’) is used as numéraire. In the latter case, the corresponding curves are strictly increasing.

(ii). The curves for the year 2000 are strictly decreasing, except for the cases where the product 37 or the gross output is used as numéraire. When the product 37 (the gross output) is used as numéraire, the corresponding curve is strictly increasing (alternately increasing and decreasing).

(iii). The curves for the year 2001 are strictly decreasing, except for the cases where the product 25 (‘Rubber and plastic products’) or 26 (‘Other non-metallic mineral products’) or 37 or gross output or net output is used as numéraire. In the latter cases, the corresponding curves are alternately increasing and decreasing.

Thus, we obtain strictly decreasing curves in about 96.25% of the tested cases. Additionally, we observe that when the product 37, which is the ‘primary’ product of the ‘Recycling’ industry, is used as numéraire, we derive non-strictly decreasing curves for all the tested cases.[28]

Curvature

(i). The curves for the year 1995 are convex for the whole interval of r.

(ii). The curves for the year 1997 are convex for the whole interval of r, except for the case where the product 60 (Land transport; transport via pipeline services) is used as numéraire. In the latter case, the curve switches from convex to concave at $r = 0.075$.

(iii). The investigation of the curves for the year 2000 gives (a) 38 curves which switch from convex to concave; (b) 10 curves which are concave for the whole interval of r; (c) 8 curves which switch from concave to convex and re-switch to concave; and (d) 2 curves which are convex for the whole interval of r.

(iv). The investigation of the curves for the year 2001 gives (a) 40 curves which switch from concave to convex and re-switch to concave; (b) 12 curves which are concave for the whole interval of r; (c) 1 curve which switches from convex to

[28] As it has been argued, Recycling is an activity that would not exist if there were no joint products (see Steedman (1984)). In this regard, it is worth mentioning that the ‘secondary’ production of the Recycling industry of the Finnish economy is more than 96% of the total production of that industry, whilst the total secondary production of all industries is less than 7% of the total production of the economy, for each year of our analysis.

concave; (d) 1 curve which switches from concave to convex; and (e) 1 curve which is convex for the whole interval of r.

Thus, we observe that the curves for the years 1995 and 1997 are in about 99.14% of the tested cases convex for the whole interval of r, whilst the curves for the years 2000 and 2001 alternate in curvature in about 77.88% of the tested cases. However, it has not been found any case where the curvature switches more than two times.

Linearity

The absolute value of correlation coefficient between w and r for the years 1995, 1997, 2000 and 2001 is in the range of 99.81%-99.98%, 99.89%-100.00%, 97.10%-99.89% and 93.88%-99.90%, respectively.[29] Thus, it can be said that the $w-r$ curves for the years 1995 and 1997 are 'nearly' linear, whilst we obtain curves with lesser correlation coefficients for the years 2000 and 2001.

In order to get a picture of the $w-r$ curves of the Finnish economy, in Figures 1-4 we display some of these curves.[30]

[29] Leaving aside the upward sloping $w-r$ curves, these results are not quite different from those obtained by Petrović (1991) on the basis of the SIOT of the Yugoslavian economy for the years 1976 and 1978. More specifically, Petrović found that (i) in <u>most</u> cases the curvature of the $w-r$ curves switches no more than two times; and (ii) the correlation coefficient between w and r for the year 1976 (1978) is in the range of 97.51%-99.98% (96.61%-99.99%).

[30] It should be noted that for the construction of each figure are used 10000 sample points.

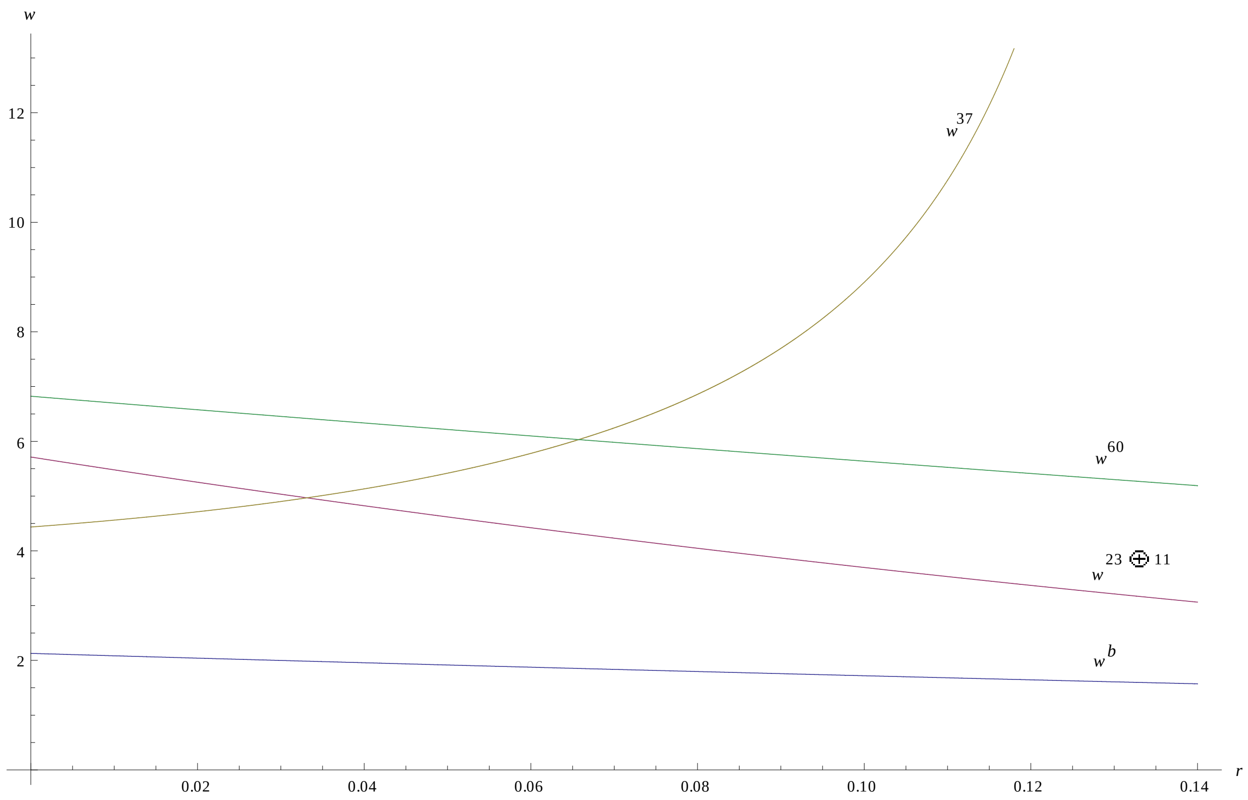

Figure 1. Wage-profit curves; Finnish economy, 1995.

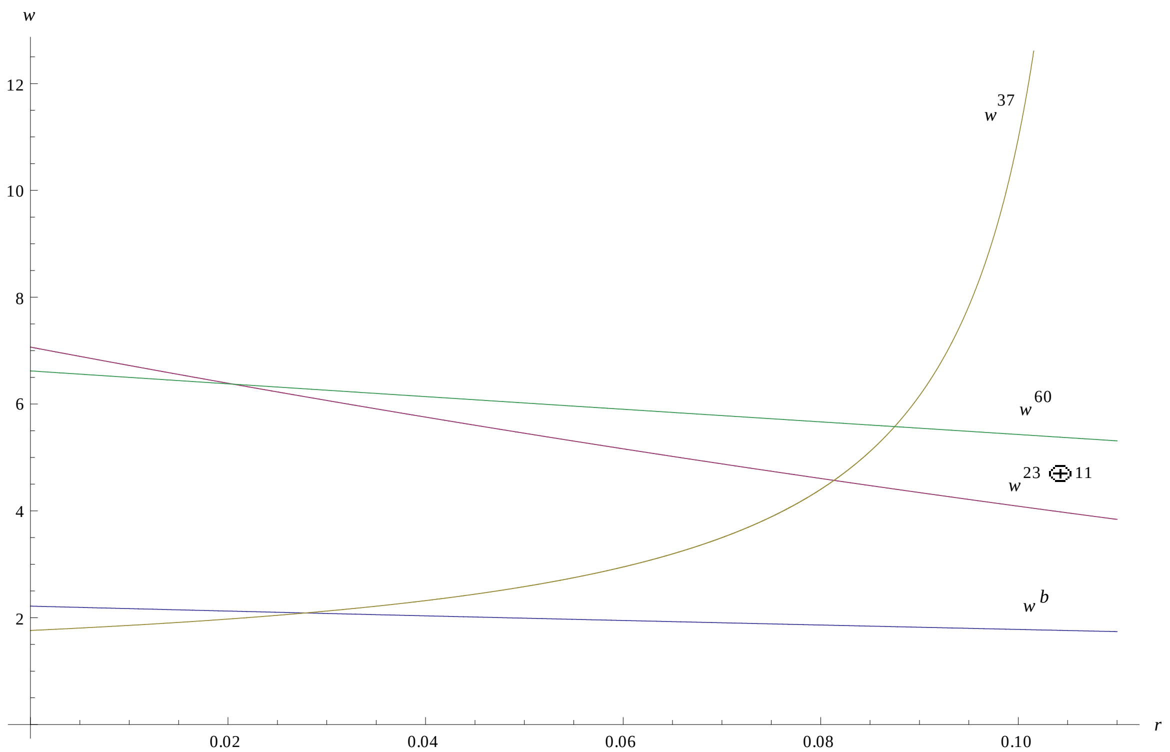

Figure 2. Wage-profit curves; Finnish economy, 1997.

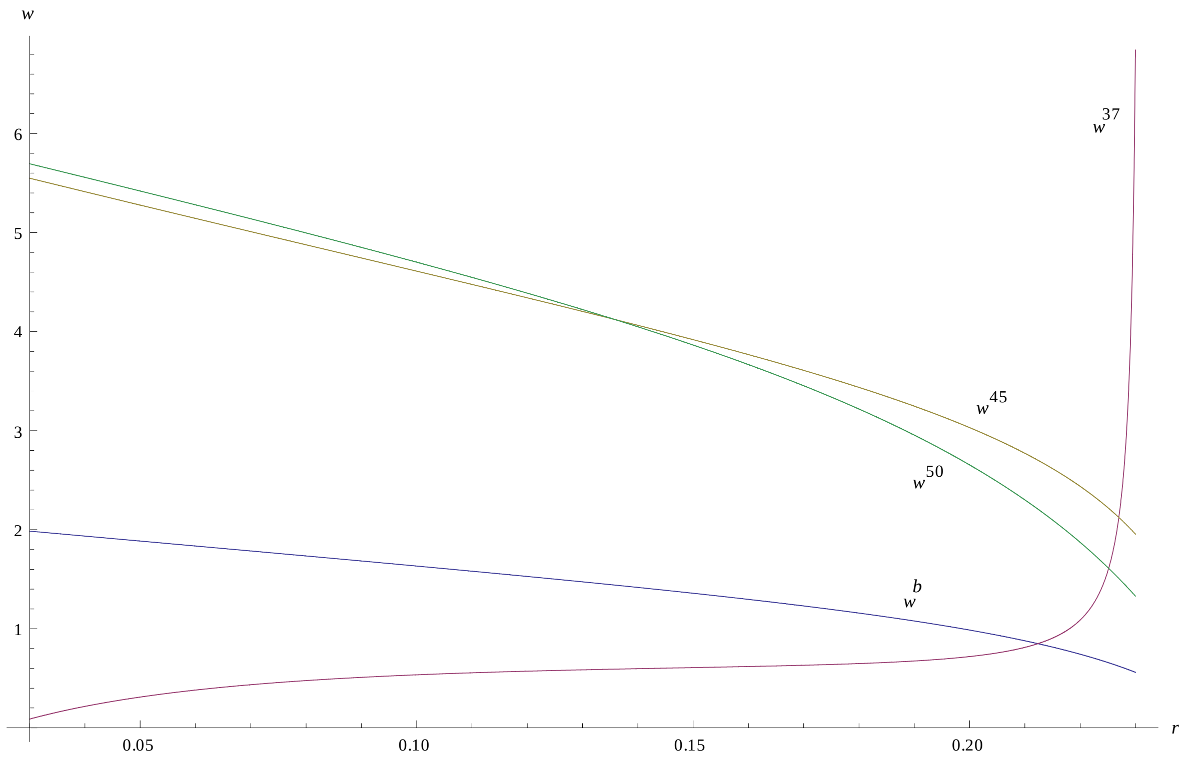

Figure 3. Wage-profit curves; Finnish economy, 2000.

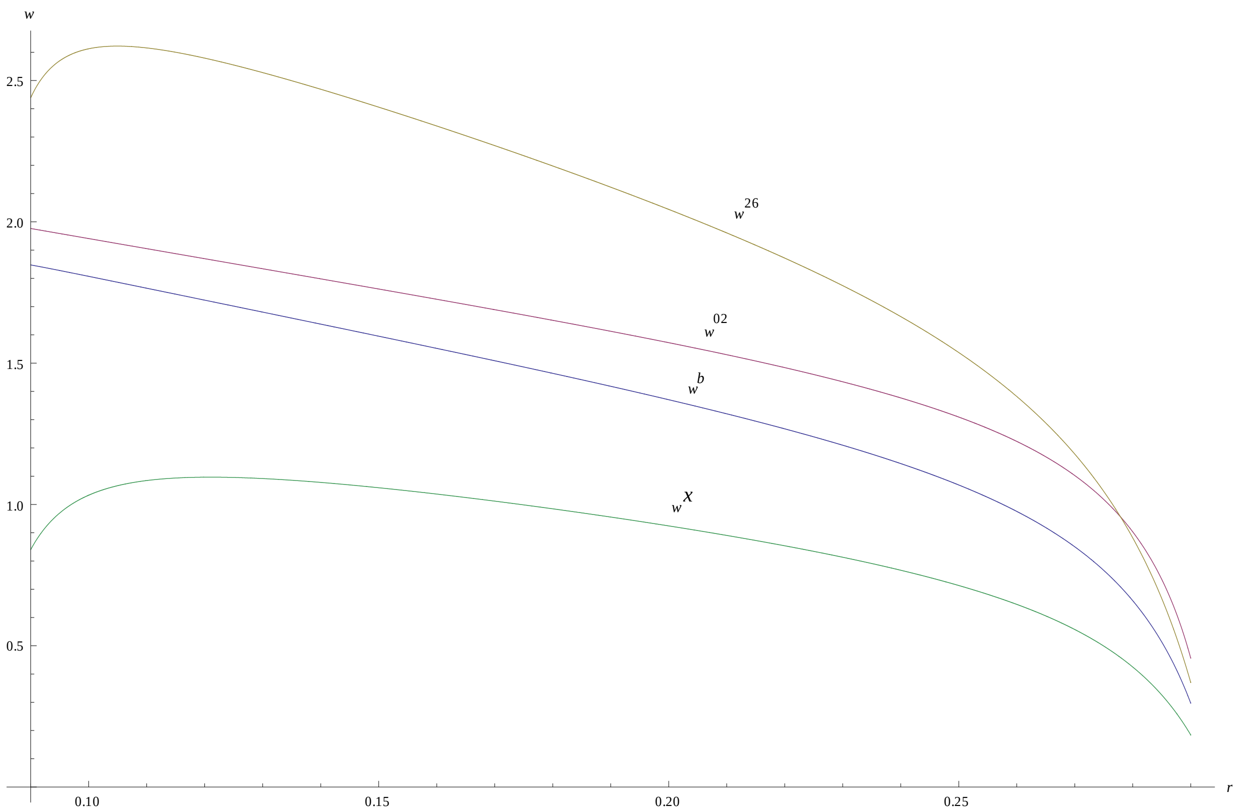

Figure 4. Wage-profit curves; Finnish economy, 2001.

In Figure 1 (2) we display the $w-r$ curves for the year 1995 (1997) associated with the numéraire (i) $23 \oplus 11$ (i.e., the product that results from the aggregation of the product 'Coke, refined petroleum products and nuclear fuels' with the product 'Crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying'); (ii) 37 (Secondary raw materials); (iii) 60 (Land transport; transport via pipeline services); and (iv) real wage rate, respectively.[31] In Figure 3 we display the $w-r$ curves for the year 2000 associated with the numéraire (i) 37; (ii) 45 (Construction work); (iii) 50 (Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel); and (iv) real wage rate, respectively. Finally, in Figure 4 we display the $w-r$ curves for the year 2001 associated with the numéraire (i) 02 (Products of forestry, logging and related services); (ii) 26 (Other non-metallic mineral products); (iii) gross output; and (iv) real wage rate, respectively.[32]

In the following, we investigate whether the above results are robust to the choice of precision in the numerical calculations. For example, we examine whether there exists an interval of $r > 0$ such that $\mathbf{P} > \mathbf{0}$ with step equal to 0.0001 (instead of 0.01) and we find that the labour commanded prices for the years 2000 and 2001 are also positive for $0.0247 \leq \mathrm{r} \leq 0.0300$ and $0.0825 \leq \mathrm{r} \leq 0.0900$, respectively (compare with the values reported in Table 1). The investigation of the monotonicity of the $w-r$ curves, within the above intervals of r, reveals that in most cases there exists an interval of r in which the $w-r$ curves are increasing. More specifically, it is found that, with the exception of the case where the commodity 14 (Other mining and quarrying products) or 66 (Insurance and pension funding services, except compulsory social security services) or 95 (Private households with employed persons) is used as numéraire, the $w-r$ curves for the year 2000 are alternately increasing and decreasing. Furthermore, with the exception of the case where the commodity 14 or 95 is used as numéraire, the $w-r$ curves for the year 2001 are

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[31] The symbol w^i denotes the wage rate expressed in terms of commodity i, where i is the CPA code of the respective commodity. Furthermore, the symbol w^b (w^x) denotes the wage rate expressed in terms of consumption basket (gross output).

[32] It is worth noting that further empirical research from the author of this paper, based on the SUT of the Greek economy (for the years 1995 through 1999), yielded similar results, i.e., it has been found that (i) the systems under consideration do not have the usual properties of single-product systems; and (ii) the monotonicity of the $w-r$ curves depends on the adopted normalization condition.

alternately increasing and decreasing. In other words, it is found that there exists an interval of the uniform rate of profits in which $54/57$ (55/57) of the labour commanded prices for the year 2000 (2001) are inversely related with the profit rate. In Figure 5 we display the $w-r$ curves for the year 2000 (for $0.0247 \leq \mathrm{r} \leq 0.0250$) associated with the numéraire (i) 13 (Metal ores); (ii) 62 (Air transport services); (iii) 72 (Computer and related services); and (iv) real wage rate, respectively, whilst in Figure 6 we display the $w-r$ curves for the year 2001 (for $0.0825 \leq \mathrm{r} \leq 0.09$) associated with the numéraire (i) 01 (Products of agriculture, hunting and related services); (ii) 15 (Food products and beverages); (iii) 40 (Electrical energy, gas, steam and hot water); and (iv) real wage rate, respectively.

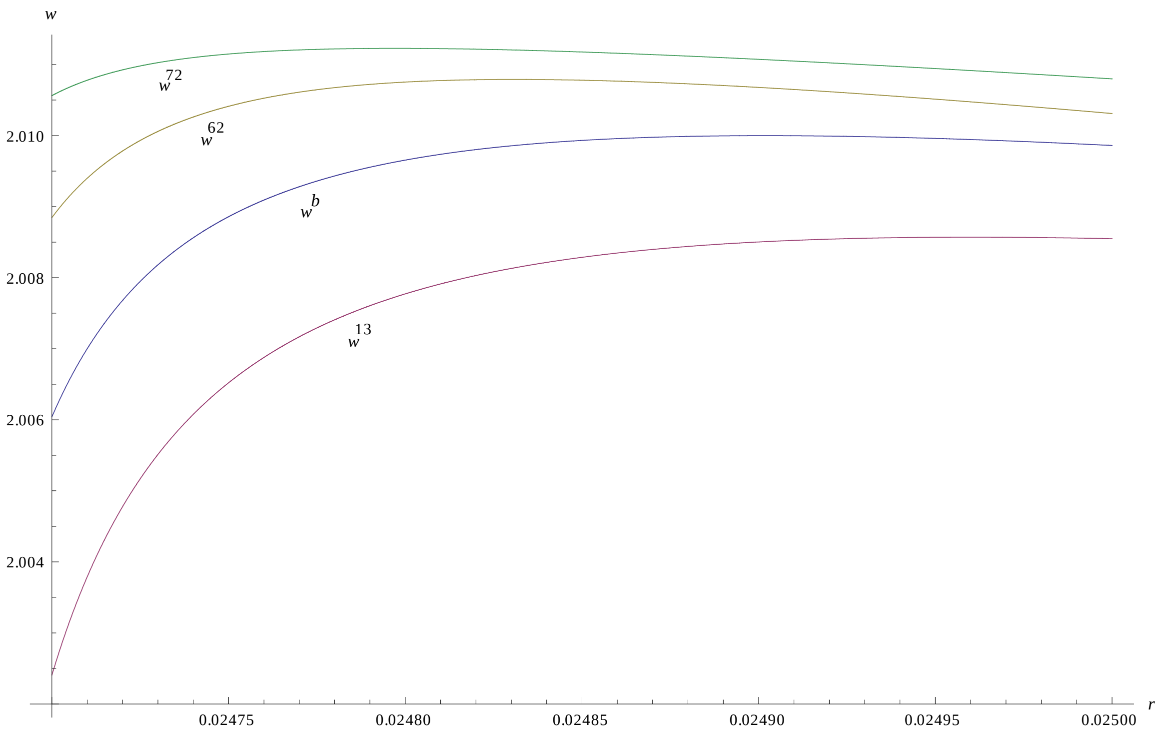

Figure 5. Wage-profit curves; Finnish economy, 2000.

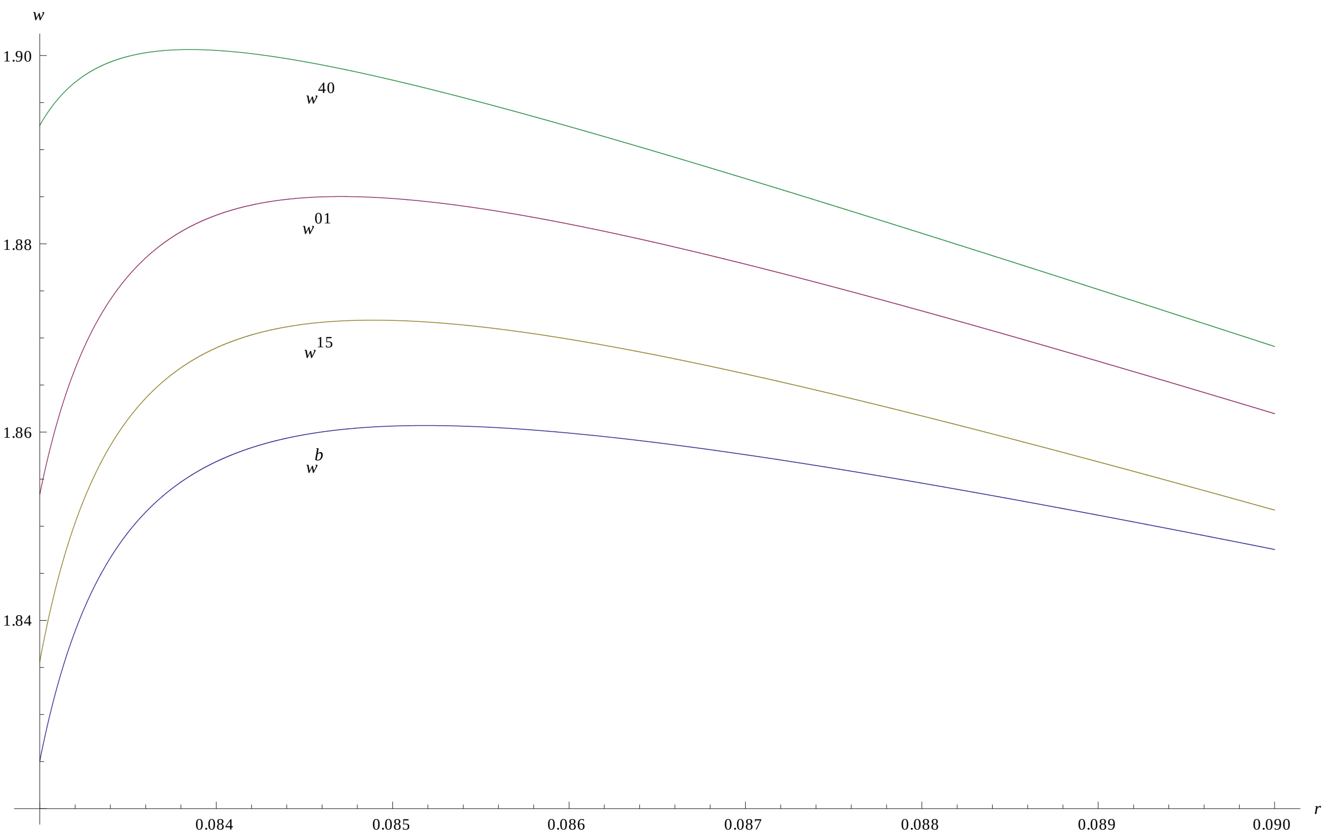

Figure 6. Wage-profit curves; Finnish economy, 2001.

Thus, it follows that the empirical results can be extremely sensitive to the choice of precision in calculations.

5. CONCLUDING REMARKS

The exploration of the shape of wage-profit curves on the basis of a usual linear model of joint production and data from the Supply and Use Tables of the Finnish economy (for the years 1995 through 2004) gave the following results:

(i). The systems under consideration are not 'all-productive' and, therefore, they do not have the properties of a single-product system. Furthermore, the systems are not 'r-all-engaging' and, therefore, does not exist an interval of the uniform rate of profits in which the economy behaves as indecomposable single-product systems.

(ii). There exists an interval of the uniform rate of profits in which the 'labour commanded' prices for the years 1995 through 1998 and 2000 through 2002 are positive, i.e., 0%-14% (1995), 0%-1% (1996), 0%-11% (1997), 0%-5% (1998), 3%-

23% (2000), 9%-29% (2001) and 25%-26% (2002), respectively. On the other hand, it has not been found such an interval for the years 1999, 2003 and 2004.

(iii). Although the monotonicity of the $w-r$ curves (for the years 1995, 1997, 2000 and 2001) depends on the adopted normalization condition, we obtain strictly decreasing $w-r$ curves in about 96.25% of the tested cases. The downward sloping $w-r$ curves for the years 1995 and 1997 are 'nearly' linear (i.e., the absolute value of correlation coefficient between w and r is in the range of 99.81%-99.98% and 99.89%-100.00%, respectively), whilst we obtain curves with lesser correlation coefficients for the years 2000 and 2001 (i.e., 97.10%-99.89% and 93.88%-99.90%, respectively). Moreover, the $w-r$ curves for the years 1995 and 1997 are in about 99.14% of the tested cases convex for the whole interval of the uniform rate of profits, whilst the curves for the years 2000 and 2001 alternate in curvature in about 77.88% of the tested cases.

(iv). Finally, it has been found that the empirical results for the years 2000 and 2001 are sensitive to the choice of precision in calculations. More specifically, an increase of the chosen precision reveals that there exists an interval of the uniform rate of profits in which $54/57$ (55/57) of the labour commanded prices for the year 2000 (2001) are inversely related with the profit rate. Thus, the corresponding $w-r$ curves are upward sloping.

Since in the real world joint production processes are by no means rare, these findings would seem to be of some importance. In any case, however, they tend to weaken the central conclusions of the relevant empirical studies (based on single-product systems), that is: 'First, the theoretical results that rely on pronounced curvature of wage-profit curves are not empirically sound. In particular much discussed phenomenons of capital reversal and double switching turn out to be highly improbable events in any actual economy. Second, although the mentioned propositions advanced by Ricardo, Marx and neoclassical economics are not true in general, they appear to be good approximation to reality' (Petrović, 1991, p. 108). Even without referring to the empirical importance of reswitching and reverse capital deepening, the findings of this study cast doubt on whether the neoclassical theory and/or the Marxian labour theory of value can be considered as a good approximation of actual economic systems, in the sense that the existence of upward sloping wage-profit curves contradicts their inner logic.

Future research efforts should (i) investigate whether the findings of this study are representative of actual economic systems, or constitute exceptional cases, by using input-output data from various countries; (ii) concretize the model by including the presence of fixed capital and the degree of its utilization, depreciation, turnover times, taxes and subsidies; and (iii) try to explain the quasi-linearity of the downward sloping $w-r$ curves.

APPENDIX: A NOTE ON THE DATA

The SUT of the Finnish economy and the corresponding levels of sectoral employment are provided via the Eurostat website http://ec.europa.eu/eurostat. The SUT describe 59 products, which are classified according to CPA (Classification of Product by Activity) and 59 industries, which are classified according to NACE (General Industrial Classification of Economic Activities within the European Communities). The described products and their correspondence to CPA are reported in Table A1 below. However, all the elements associated with the product 12 (Uranium and thorium ores) and industry 12 (Mining of uranium and thorium ores) equal zero and, therefore, we remove them from our analysis. Furthermore, all the elements associated with the product 11 (Crude petroleum and natural gas) and industry 11 (Extraction of crude petroleum and natural gas) in the Make matrices (i.e., the part of the Supply Tables that describes domestic production) equal zero, and, therefore, we remove them from our analysis, whilst there are elements associated with the product 11 in the Use matrices (i.e., the part of the Use Tables that describes intermediate consumption) that are positive. In order to derive 'square' Make and Use matrices, we aggregate the product 11 with the 'primary product' (Coke, refined petroleum products and nuclear fuels) of industry 23 (Manufacture of coke, refined petroleum products and nuclear fuels). This choice is based on the fact that the product 11 is mainly used by the industry 23. Thus, we derive Make and Use matrices of dimensions 57×57.

All the SUT used in our analysis are at current 'basic prices'. It is important to note that we decided to use Finland's SUT mainly because there were available Use Tables at basic prices. Since Use Tables are originally constructed at 'purchasers' prices', most statistical offices do not provide these tables at 'basic prices'. The market prices of all products are taken to be equal to one; that is to say, the physical unit of measurement of each product is that unit which is worth of a monetary unit (see, e.g., Miller and Blair, 1985, p. 356). Wage differentials are used to homogenize the sectoral employment (see, e.g., Sraffa, 1960, §10, and Kurz and Salvadori, 1995, pp. 322-325), i.e., the j-th element of the vector of employment levels process by process, $\mathbf{a}$, is determined as follows: $a_j = \Lambda_j (w_j^{\mathrm{M}} / w_{\min}^{\mathrm{M}})$, where Λ_j and w_j^{M} are total employment and money wage rate, in terms of market prices, of the j-th sector, respectively, and $w_{\min}^{\mathrm{M}}$ is the minimum sectoral money wage rate in terms of market prices. Alternatively, the homogenization of employment could be achieved, for example, through the economy's average wage; in fact, the empirical results are robust to alternative normalizations with respect to homogenization of labour inputs. Furthermore, by assuming that workers do not save and that their consumption has the same composition as the vector of private households consumption expenditure, $\mathbf{c}$, directly available in the SUT, the vector of the real wage rate, $\mathbf{b}$, is determined as follows: $\mathbf{b} = (w_{\min}^{\mathrm{M}} / \mathbf{ec})\mathbf{c}$, where $\mathbf{e} \equiv [1,1,...,1]$ represents the vector of market prices (see also, e.g., Okishio and Nakatani, 1985, pp. 66-67). Finally, it should be noted that in the available SUT of the Finnish economy we do not have data on the matrix of fixed capital coefficients and the non-competitive imports. As a result, our investigation is based on a model for a closed economy with circulating capital.

Table A1. Product classification.

No	CPA	Nomenclature
1	01	Products of agriculture, hunting and related services
2	02	Products of forestry, logging and related services
3	05	Fish and other fishing products; services incidental of fishing
4	10	Coal and lignite; peat
5	11	Crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying
6	12	Uranium and thorium ores
7	13	Metal ores
8	14	Other mining and quarrying products
9	15	Food products and beverages
10	16	Tobacco products
11	17	Textiles
12	18	Wearing apparel; furs
13	19	Leather and leather products
14	20	Wood and products of wood and cork (except furniture); articles of straw and plaiting materials
15	21	Pulp, paper and paper products
16	22	Printed matter and recorded media
17	23	Coke, refined petroleum products and nuclear fuels
18	24	Chemicals, chemical products and man-made fibres

19	25	Rubber and plastic products
20	26	Other non-metallic mineral products
21	27	Basic metals
22	28	Fabricated metal products, except machinery and equipment
23	29	Machinery and equipment n.e.c.
24	30	Office machinery and computers
25	31	Electrical machinery and apparatus n.e.c.
26	32	Radio, television and communication equipment and apparatus
27	33	Medical, precision and optical instruments, watches and clocks
28	34	Motor vehicles, trailers and semi-trailers
29	35	Other transport equipment
30	36	Furniture; other manufactured goods n.e.c.
31	37	Secondary raw materials
32	40	Electrical energy, gas, steam and hot water
33	41	Collected and purified water, distribution services of water
34	45	Construction work
35	50	Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel
36	51	Wholesale trade and commission trade services, except of motor vehicles and motorcycles
37	52	Retail trade services, except of motor vehicles and motorcycles; repair services of personal and household goods
38	55	Hotel and restaurant services
39	60	Land transport; transport via pipeline services
40	61	Water transport services
41	62	Air transport services
42	63	Supporting and auxiliary transport services; travel agency services
43	64	Post and telecommunication services
44	65	Financial intermediation services, except insurance and pension funding services
45	66	Insurance and pension funding services, except compulsory social security services
46	67	Services auxiliary to financial intermediation
47	70	Real estate services
48	71	Renting services of machinery and equipment without operator and of personal and household goods
49	72	Computer and related services
50	73	Research and development services
51	74	Other business services
52	75	Public administration and defence services; compulsory social security services
53	80	Education services
54	85	Health and social work services
55	90	Sewage and refuse disposal services, sanitation and similar services
56	91	Membership organisation services n.e.c.
57	92	Recreational, cultural and sporting services
58	93	Other services
59	95	Private households with employed persons

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